Stochastic Mean Field (SMF) description

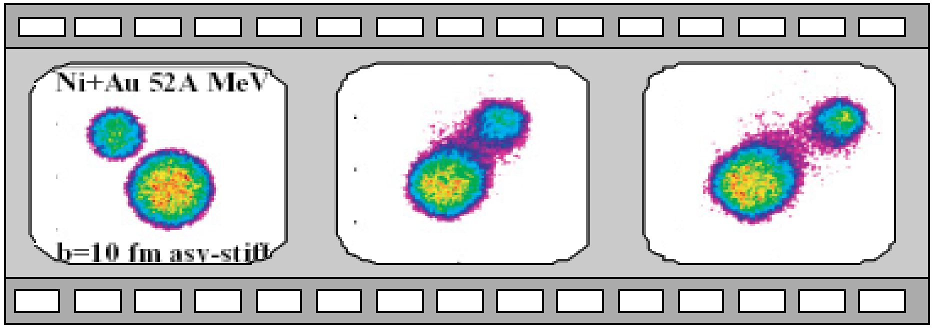
TRANSPORT 2017

March 27-30, 2017 **FRIB-MSU, East Lansing, Michigan, USA**

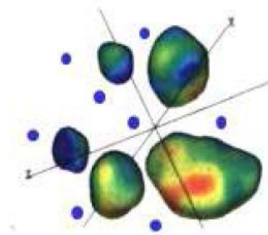
Maria Colonna

INFN - Laboratori Nazionali del Sud (Catania)

Dynamics of many-body system I



- Mean-field (one-body) dynamics
- Two-body correlations
- Fluctuations



one-body density matrix

two-body density matrix

$$\rho_2(12,1'2') = \underbrace{\rho_1(1,1')\rho_1(2,2')}_{\text{one-body}} + \delta\sigma(12,1'2')$$

$$H = H_0 + v_{1,2}$$

Mean-field ***Residual interaction***

$$\boxed{i\hbar\frac{\partial}{\partial t}\rho_1(1,1',t) = \langle 1|[H_0,\rho_1(t)]|1'\rangle + K[\rho_1] + \delta K[\rho_1,\delta\sigma]}$$

TDHF

$K = F(\rho_1, |v|^2)$ <u>*Average effect of the residual interaction*</u>

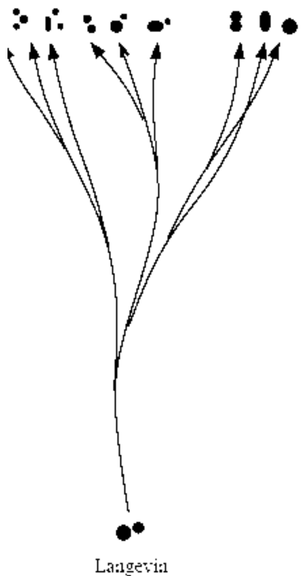
$\delta K = F(v, \delta\sigma)$ $\quad < \delta K > = 0 \quad < \delta K \delta K > \longrightarrow$ <u>*Fluctuations*</u>

Dynamics of many-body systems II

Collision Integral $$\boldsymbol{K} = g\sum_{234} W(12;34)\left[\bar{f}_1\bar{f}_2 f_3 f_4 - f_1 f_2 \bar{f}_3\bar{f}_4\right]$$

$$\bar{f} = 1 - f$$

Transition rate W interpreted in terms of NN cross section



-- If statistical fluctuations larger than quantum ones

$$< \delta K(p,t)\delta K(p',t') > = C\delta(t-t')$$

$$C(\boldsymbol{p}_a,\boldsymbol{p}_b,\boldsymbol{r},t) = \delta_{ab}\sum_{234} W(a2;34)F(a2;34)$$

$$F(12;34) \equiv f_1 f_2 \bar{f}_3\bar{f}_4 + \bar{f}_1\bar{f}_2 f_3 f_4.$$

Main ingredients:

- **Residual interaction (2-body correlations and fluctuations)**
  In-medium nucleon cross section
- **Effective interaction**
  (self consistent mean-field)

Skyrme, Gogny forces

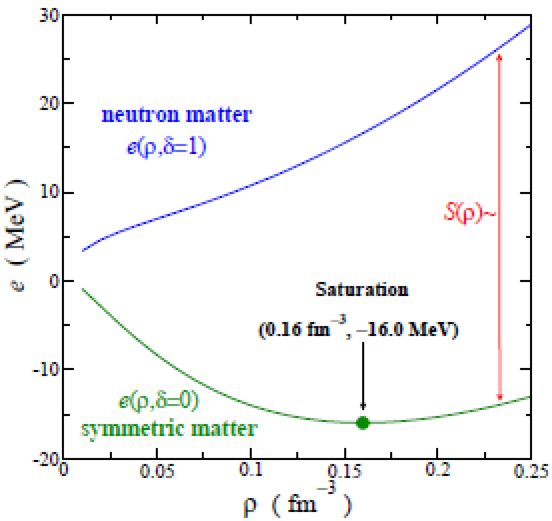
- **Effective interactions**

Energy Density Functional theories: The exact density functional is approximated with powers and gradients of one-body nucleon densities and currents.

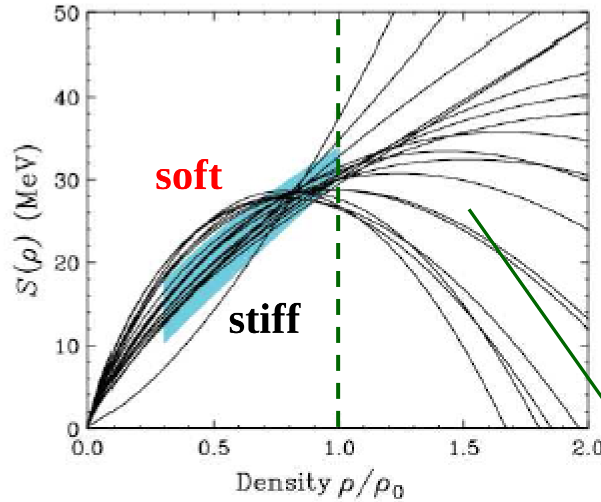
$$E = \left\langle \Psi \middle| \hat{H} \middle| \Psi \right\rangle \approx \left\langle \Phi \middle| \hat{H}_{eff} \middle| \Phi \right\rangle = E[\hat{\rho}]$$

The nuclear **Equation of State (T = 0)**

Energy per nucleon E/A (MeV)



Symmetry energy E_{sym} (MeV)



poorly known …

predictions of several effective interactions

$$\frac{E}{A}(\rho,\beta) = \frac{E}{A}(\rho,\beta=0) + E_{sym}(\rho)\beta^2 + O(\beta^4)$$

symm. matter ***symm. energy***

expansion around normal density

$$E_{sym}(\rho) = S_0 + L\frac{\rho-\rho_0}{3\rho_0} + \ldots$$

or J

β = *asymmetry parameter* = $(\rho_n - \rho_p)/\rho$

➢ analogy with **Weizsacker mass formula** for nuclei (symmetry term) !

$25 \le J \le 35$ MeV $\quad 20 \le L \le 120$ MeV

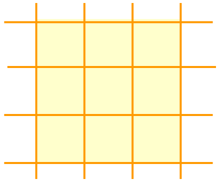
➢1. *Semi-classical approximation to Nuclear Dynamics*

Chomaz,Colonna, Randrup
Phys. Rep. 389 (2004)
Baran,Colonna,Greco, Di Toro
Phys. Rep. 410, 335 (2005)

Transport equation for the one-body distribution function f

Semi-classical analog of the Wigner transform of the one-body density matrix

Density $f = f(\mathbf{r},\mathbf{p},t)$



Phase space (***r***,***p***)

➢ $$\boxed{\frac{df(r,p,t)}{dt} = \frac{\partial f(r,p,t)}{\partial t} + \{f, H_0\} = 0}$$

$H_0 = \mathrm{T} + \mathrm{U}$

Vlasov Equation,
like Liouville equation:
The phase-space density is constant in time

The mean-fiels potential U is self-consistent: **U = U(ρ)**
Nucleons move in the field created by all other nucleons

Semi-classical approximation $\longrightarrow$ **transport theories**

$$\frac{df(r,p,t)}{dt} = \underbrace{\frac{\partial f(r,p,t)}{\partial t} + \{f, h\}}_{\text{Vlasov}} = k_I[f] + \delta k$$

Boltzmann-Langevin

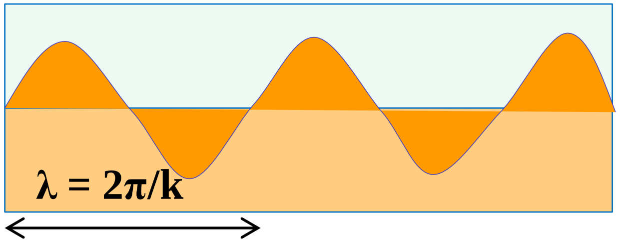
Correlations,
Fluctuations

From BOB to SMF ……

- Fluctuations from ***external stochastic*** force (tuning of the most unstable modes)

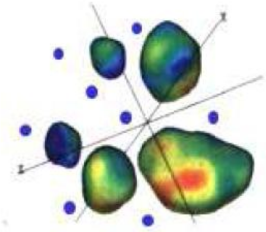
$$\dot{f} = \bar{I}[f] + \frac{\partial \mathcal{U}_{ext}}{\partial \mathbf{r}} \frac{\partial f}{\partial \mathbf{p}}$$

Brownian One Body (**BOB**) dynamics



Chomaz,Colonna,Guarnera,Randrup PRL73,3512(1994)

multifragmentation event

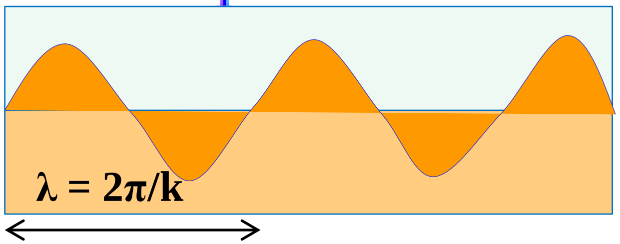


From BOB to SMF ……

- Fluctuations from ***external*** stochastic force (tuning of the most unstable modes)

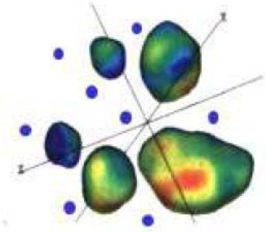
$$\dot{f} = \bar{I}[f] + \frac{\partial U_{ext}}{\partial r}\frac{\partial f}{\partial p}$$

Brownian One Body (**BOB**) dynamics

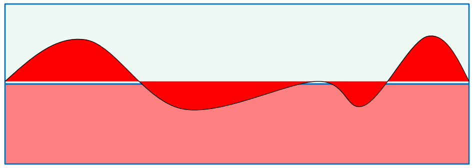


Chomaz,Colonna,Guarnera,Randrup
PRL73,3512(1994)

multifragmentation event



- Stochastic Mean-Field (**SMF**) model :
  Thermal fluctuations (at local equilibrium) are projected on the coordinate space by agitating the spacial density profile



M.Colonna et al., NPA642(1998)449

Details of the model

- $$f(\mathbf{r},\mathbf{p},t) = \frac{Ch^3}{4}\sum_i g_r(\mathbf{r}-\mathbf{r}_i)g_p(\mathbf{p}-\mathbf{p}_i),$$

  triangular functions in $\mathbf{r}$ space (for g_r) and δ functions in momentum space (for g_p)

  triangular function $g(x^j - x_i^j) = 2l - |x^j - x_i^j|$ $\quad$ $l = \mathbf{1}$ fm

  lattice size

  Total number of test particles: $N_{tot} = N_{test} * A$

- **System total energy (lattice Hamiltonian):**

$$E_{tot} = \sum_i p_i^2/(2m) + N_{test}[\int d\mathbf{r}\ \rho(\mathbf{r})E_{pot}(\rho_n,\rho_p)$$

$$+ \int d\mathbf{r}\ \rho_p(\mathbf{r})E_{pot}^{Coul}(\rho_p)/2,\ ]$$

Potential energy

$$E_{pot}(\rho) = \frac{A}{2}\tilde{\rho} + \frac{B}{\sigma+1}\tilde{\rho}^{\sigma} + \frac{C_{surf}}{2\rho}(\nabla\rho)^2 + \frac{1}{2}C_{sym}(\rho)\tilde{\rho}\beta^2,$$

where $\tilde{\rho} = \rho/\rho_0$ (ρ_0 denotes the saturation density), $A = -356\ MeV$, $B = 303\ MeV$, $\sigma = 7/6$. β = asymmetry parameter

- $C_{surf} = -6/\rho_0^{5/3}\ MeV\ fm^5.$

Negative surface term to correct surface effects induced by the the use of finite width t.p. packets

Symmetry energy parametrizations :

asy-EoS	E_{sym}/A	L(MeV)
asysoft	30.	14.
asystiff	28.	73.
asysupstiff	28.	97.

New Skyrme interactions (SAMi-J family) recently introduced

Hua Zheng et al.
PHYSICAL REVIEW C **94**, 014313 (2016)

- the Coulomb potential is determined solving the Laplace equation:

$$\nabla^2 E_{pot}^{Coul} = -4\pi e^2 \rho_p = -18.1\rho_p$$

$\longrightarrow$ *Initialization and dynamical evolution*

- Ground state initialization with Thomas-Fermi

- Test particle positions and momenta are propagated according to the Hamilton equations (non relativistic)

Details of the model: Collision Integral

- **Mean free path method :**

  each test particle has just one collision partner

$$\tau_{col} = \frac{\lambda}{v_{kl}} = \frac{1}{\rho \sigma_{NN} v_{kl}}, \qquad P_{col}(\Delta t) \approx \Delta t / \tau_{col}$$

Δt = time step

- blocking factors, defined as $P_{Pauli} = (1 - f_l)(1 - f_k)$,

we now take a Θ function in $\mathbf{r}$ space and a gaussian function, with $\sigma = 29\ MeV/c$, in momentum space. The Θ function is defined as: $\Theta(\mathbf{r} - \mathbf{r}_i) = \Theta(R - |\mathbf{r} - \mathbf{r}_i|)$, with $R = 2.53 fm$. The new definition makes the occupation number smoother (though less local), reducing fluctuations which may induce spurious collisions.

- Free n-p and p-p cross sections, with a maximum cutoff of 50 mb

Fluctuation tuning

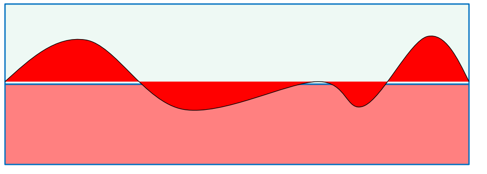
When local equilibrium is achieved:

- $$\sigma_f^2 = f(1-f)$$ → Fluctuation variance for a fermionic system at equilibrium

$$\sigma_\rho^2(\mathbf{r},t) = \frac{1}{V}\int \frac{d\mathbf{p}}{h^3/4}\sigma_f^2(\mathbf{r},\mathbf{p},t).$$

$$\sigma_\rho^2 = \frac{\rho}{V}\frac{3T}{2\varepsilon_F}[1 - \frac{\pi^2}{12}(\frac{T}{\epsilon_F})^2 + ...].$$

- Stochastic Mean-Field (**SMF**) model :
  Fluctuations are projected on the coordinate space
  by agitating the spacial density profile



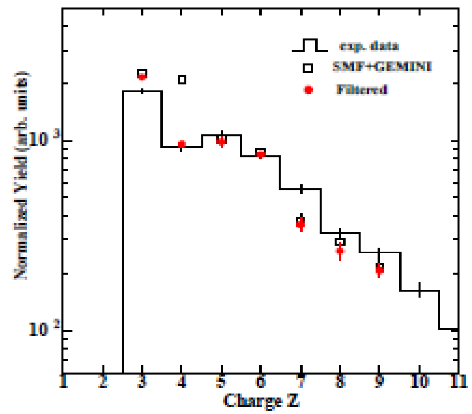
[1] V.Baran, M.Colonna, V.Greco and M.Di Toro, Phys. Rep. **410**, 335 (2005)
[2] M.Colonna et al., Nucl. Phys. A **642**, 449 (1998)

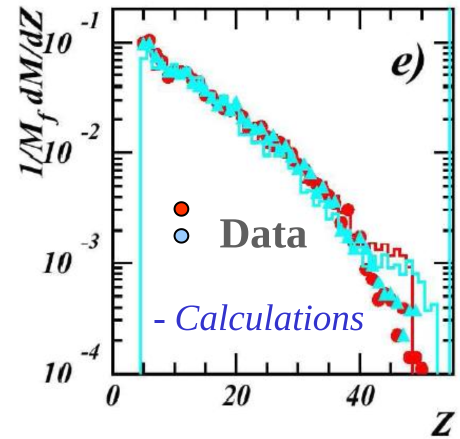
Ph.Chomaz, M.Colonna, J.Randrup, Phys. Rep. **389**, 263 (2004)

Some applications ……

Charge distribution

E. De Filippo et al., PRC(2012)

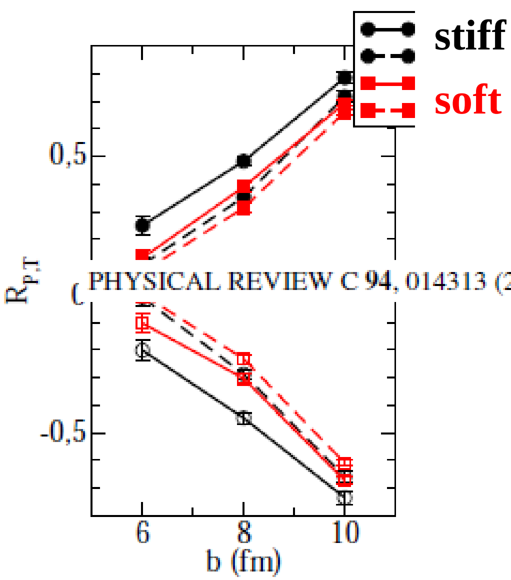




J.Frankland et al., NPA 2001

- ❑ **Fragmentation studies in central and semi-peripheral collisions**

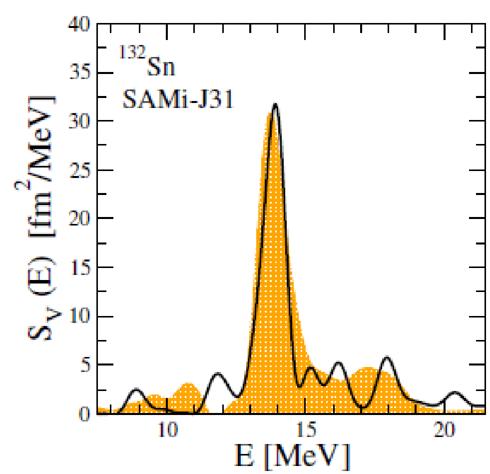
- ❑ **Isospin effects at Fermi energies**



PHYSICAL REVIEW C 94, 014313 (2016)

J. Rizzo et al., NPA(2008)

- ❑ **Small amplitude dynamics (collective modes) and low-energy reaction dynamics**



Hua Zheng et al.

PHYSICAL REVIEW C 94, 014313 (2016)